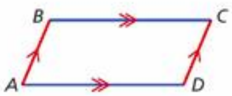
Who uses this?

Race car designers can use a parallelogram-shaped linkage to keep the wheels of the car vertical on uneven surfaces. (See Example 1.)

Any polygon with four sides is a quadrilateral. However, some quadrilaterals have special properties. These *special quadrilaterals* are given their own names.

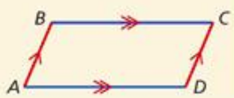
A quadrilateral with two pairs of parallel sides is a **parallelogram**. To write the name of a parallelogram, you use the symbol ▱.

Parallelogram *ABCD*
▱*ABCD*



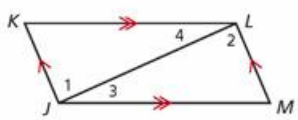
$\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$

Theorem 6-2-1 Properties of Parallelograms

THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is a parallelogram, then its opposite sides are congruent. (▱ → opp. sides ≅)		$\overline{AB} \cong \overline{CD}$ $\overline{BC} \cong \overline{DA}$

Theorem 6-2-1

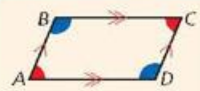
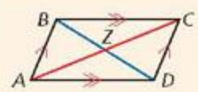
Given: *JKLM* is a parallelogram.
Prove: $\overline{JK} \cong \overline{LM}$, $\overline{KL} \cong \overline{MJ}$



Proof:

Statements	Reasons
1. *JKLM* is a parallelogram.	**1.** Given
2. $\overline{JK} \parallel \overline{LM}$, $\overline{KL} \parallel \overline{MJ}$	**2.** Def. of ▱
3. ∠1 ≅ ∠2, ∠3 ≅ ∠4	**3.** Alt. Int. ∠s Thm.
4. $\overline{JL} \cong \overline{JL}$	**4.** Reflex. Prop. of ≅
5. △*JKL* ≅ △*LMJ*	**5.** ASA *Steps 3, 4*
6. $\overline{JK} \cong \overline{LM}$, $\overline{KL} \cong \overline{MJ}$	**6.** CPCTC

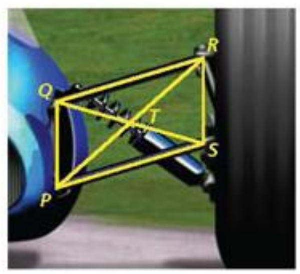
Theorems — **Properties of Parallelograms**

	THEOREM	HYPOTHESIS	CONCLUSION
6-2-2	If a quadrilateral is a parallelogram, then its opposite angles are congruent. ($\square \rightarrow$ opp. $\measuredangle$s $\cong$)		$\angle A \cong \angle C$ $\angle B \cong \angle D$
6-2-3	If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. ($\square \rightarrow$ cons. $\measuredangle$s supp.)		$m\angle A + m\angle B = 180°$ $m\angle B + m\angle C = 180°$ $m\angle C + m\angle D = 180°$ $m\angle D + m\angle A = 180°$
6-2-4	If a quadrilateral is a parallelogram, then its diagonals bisect each other. ($\square \rightarrow$ diags. bisect each other)		$\overline{AZ} \cong \overline{CZ}$ $\overline{BZ} \cong \overline{DZ}$

You will prove Theorems 6-2-3 and 6-2-4 in Exercises 45 and 44.

E 1 *Racing Application*

The diagram shows the parallelogram-shaped linkage that joins the frame of a race car to one wheel of the car. In $\square PQRS$, $QR = 48$ cm, $RT = 30$ cm, and $m\angle QPS = 73°$. Find each measure.



A PS

$\overline{PS} \cong \overline{QR}$ — *$\square \rightarrow$ opp. sides $\cong$*

$PS = QR$ — *Def. of $\cong$ segs.*

$PS = 48$ cm — *Substitute 48 for QR.*

B $m\angle PQR$

$m\angle PQR + m\angle QPS = 180°$ — *$\square \rightarrow$ cons. $\measuredangle$s supp.*

$m\angle PQR + 73 = 180$ — *Substitute 73 for $m\angle QPS$.*

$m\angle PQR = 107°$ — *Subtract 73 from both sides.*

C PT

$\overline{PT} \cong \overline{RT}$ — *$\square \rightarrow$ diags. bisect each other*

$PT = RT$ — *Def. of $\cong$ segs.*

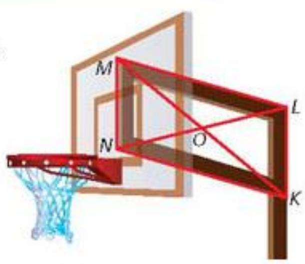
$PT = 30$ cm — *Substitute 30 for RT.*

CHECK IT OUT! **In $\square KLMN$, $LM = 28$ in., $LN = 26$ in., and $m\angle LKN = 74°$. Find each measure.**

1a. KN

1b. $m\angle NML$

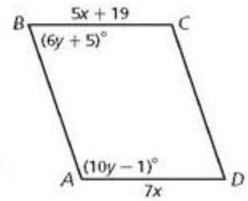
1c. LO



EXAMPLE 2 **Using Properties of Parallelograms to Find Measures**

Algebra

ABCD is a parallelogram. Find each measure.



A *AD*

$\overline{AD} \cong \overline{BC}$	*▱ → opp. sides ≅*
$AD = BC$	*Def. of ≅ segs.*
$7x = 5x + 19$	*Substitute the given values.*
$2x = 19$	*Subtract 5x from both sides.*
$x = 9.5$	*Divide both sides by 2.*

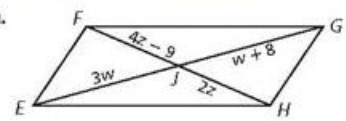
$AD = 7x = 7(9.5) = 66.5$

B $m\angle B$

$m\angle A + m\angle B = 180°$	*▱ → cons. ∠s supp.*
$(10y - 1) + (6y + 5) = 180$	*Substitute the given values.*
$16y + 4 = 180$	*Combine like terms.*
$16y = 176$	*Subtract 4 from both sides.*
$y = 11$	*Divide both sides by 16.*

$m\angle B = (6y + 5)° = [6(11) + 5]° = 71°$

CHECK IT OUT! *EFGH* is a parallelogram. Find each measure.



2a. *JG*

2b. *FH*

EXAMPLE 3 **Parallelograms in the Coordinate Plane**

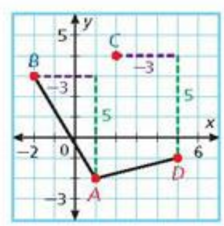
Three vertices of ▱*ABCD* are $A(1, -2)$, $B(-2, 3)$, and $D(5, -1)$. Find the coordinates of vertex *C*.

Since *ABCD* is a parallelogram, both pairs of opposite sides must be parallel.

Remember!

When you are drawing a figure in the coordinate plane, the name *ABCD* gives the order of the vertices.

Step 1 Graph the given points.



Step 2 Find the slope of $\overline{AB}$ by counting the units from *A* to *B*.
The rise from −2 to 3 is 5.
The run from 1 to −2 is −3.

Step 3 Start at *D* and count the same number of units.
A rise of 5 from −1 is 4.
A run of −3 from 5 is 2. Label (2, 4) as vertex *C*.

Step 4 Use the slope formula to verify that $\overline{BC} \parallel \overline{AD}$.

$$\text{slope of } \overline{BC} = \frac{4 - 3}{2 - (-2)} = \frac{1}{4}$$

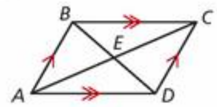
$$\text{slope of } \overline{AD} = \frac{-1 - (-2)}{5 - 1} = \frac{1}{4}$$

The coordinates of vertex *C* are (2, 4).

CHECK IT OUT! **3.** Three vertices of ▱*PQRS* are $P(-3, -2)$, $Q(-1, 4)$, and $S(5, 0)$. Find the coordinates of vertex *R*.

EXAMPLE 4 **Using Properties of Parallelograms in a Proof**

Write a two-column proof.



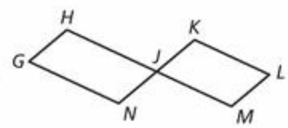
A Theorem 6-2-2

Given: *ABCD* is a parallelogram.

Prove: ∠*BAD* ≅ ∠*DCB*, ∠*ABC* ≅ ∠*CDA*

Proof:

Statements	Reasons
1. *ABCD* is a parallelogram.	1. Given
2. $\overline{AB} \cong \overline{CD}$, $\overline{DA} \cong \overline{BC}$	2. ▱ → opp. sides ≅
3. $\overline{BD} \cong \overline{BD}$	3. Reflex. Prop. of ≅
4. △*BAD* ≅ △*DCB*	4. SSS *Steps 2, 3*
5. ∠*BAD* ≅ ∠*DCB*	5. CPCTC
6. $\overline{AC} \cong \overline{AC}$	6. Reflex. Prop. of ≅
7. △*ABC* ≅ △*CDA*	7. SSS *Steps 2, 6*
8. ∠*ABC* ≅ ∠*CDA*	8. CPCTC



B Given: *GHJN* and *JKLM* are parallelograms. *H* and *M* are collinear. *N* and *K* are collinear.

Prove: ∠*G* ≅ ∠*L*

Proof:

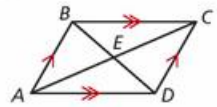
Statements	Reasons
1. *GHJN* and *JKLM* are parallelograms.	1. Given
2. ∠*HJN* ≅ ∠*G*, ∠*MJK* ≅ ∠*L*	2. ▱ → opp. ∠s ≅
3. ∠*HJN* ≅ ∠*MJK*	3. Vert. ∠s Thm.
4. ∠*G* ≅ ∠*L*	4. Trans. Prop. of ≅

CHECK IT OUT!

4. Use the figure in Example 4B to write a two-column proof.
Given: *GHJN* and *JKLM* are parallelograms.

EXAMPLE 4 Using Properties of Parallelograms in a Proof

Write a two-column proof.



A Theorem 6-2-2

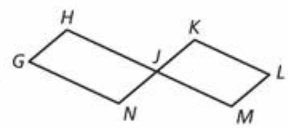
Given: *ABCD* is a parallelogram.

Prove: $\angle BAD \cong \angle DCB$, $\angle ABC \cong \angle CDA$

Proof:

Statements	Reasons
1. *ABCD* is a parallelogram.	1. Given
2. $\overline{AB} \cong \overline{CD}$, $\overline{DA} \cong \overline{BC}$	2. ▱ → opp. sides ≅
3. $\overline{BD} \cong \overline{BD}$	3. Reflex. Prop. of ≅
4. $\triangle BAD \cong \triangle DCB$	4. SSS *Steps 2, 3*
5. $\angle BAD \cong \angle DCB$	5. CPCTC
6. $\overline{AC} \cong \overline{AC}$	6. Reflex. Prop. of ≅
7. $\triangle ABC \cong \triangle CDA$	7. SSS *Steps 2, 6*
8. $\angle ABC \cong \angle CDA$	8. CPCTC

B **Given:** *GHJN* and *JKLM* are parallelograms. *H* and *M* are collinear. *N* and *K* are collinear.



Prove: $\angle G \cong \angle L$

Proof:

Statements	Reasons
1. *GHJN* and *JKLM* are parallelograms.	1. Given
2. $\angle HJN \cong \angle G$, $\angle MJK \cong \angle L$	2. ▱ → opp. ∠s ≅
3. $\angle HJN \cong \angle MJK$	3. Vert. ∠s Thm.
4. $\angle G \cong \angle L$	4. Trans. Prop. of ≅

CHECK IT OUT!

4. Use the figure in Example 4B to write a two-column proof.
Given: *GHJN* and *JKLM* are parallelograms.