

6.1 Properties and Attributes of Polygons



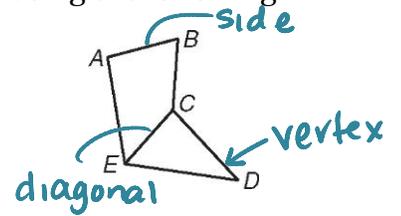
Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
15	Pentadecagon
n	n-gon

n = number of sides of the polygon

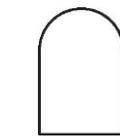
What is a polygon? a closed figure formed by 3 or more segments

Label the picture below using the following:

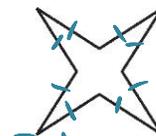
Side \overline{AB}
 Vertex $\angle D$
 Diagonal \overline{EC}



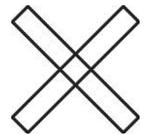
Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.



Not a polygon

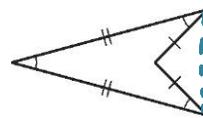


Polygon octagon

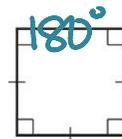


Not a polygon

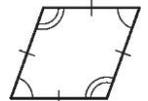
Tell whether each figure is regular or irregular.
 Tell whether each figure is concave or convex.



Irregular concave



regular convex

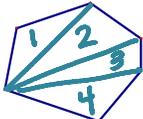


Irregular convex

Regular vs. Irregular	Concave vs. Convex
<p><u>Regular:</u></p> <ul style="list-style-type: none"> equilateral (all sides \cong) equiangular (all \angles \cong) <p><i>both</i></p>	<p><u>Concave:</u> any part of a diagonal contains pts in the exterior of the polygon</p>
<p><u>Irregular:</u></p> <p>not regular</p>	<p><u>Convex:</u> no part of the diagonal is outside the polygon</p>

Recall: The sum of the interior angles of a triangle is _____.

Polygon	# of Sides	Diagram	Number of Triangles (from one vertex)	Sum of Interior Angle Measures
Triangle	3		1	$1 \cdot 180 = 180^\circ$
Quadrilateral	4		2	$2 \cdot 180 = 360^\circ$
Pentagon	5		3	$3 \cdot 180 = 540^\circ$

Hexagon	6		4	$4 \cdot 180 = 720^\circ$
n-gon	n		$n-2$	$(n-2)180$

Polygon Interior Angle Sum Theorem:

The sum of the interior angles of a convex polygon with n sides is $S_I = (n-2)180$.

Examples:

- 1) Find the sum of the interior angle measures of a convex pentadecagon.

$$\begin{aligned}
 S_I &= (n-2)180 \\
 &= (15-2)180 \\
 &= (13)180 \\
 &= 2340^\circ
 \end{aligned}$$

- 2) Find the measure of each interior angle of a regular decagon.

$$\begin{aligned}
 \text{1st } S_I &= (n-2)180 \\
 &= (10-2)180 \\
 &= (8)180 \\
 &= 1,440^\circ
 \end{aligned}$$

$$\begin{array}{r}
 \text{2nd SUM} \\
 \hline
 \# \text{ of } \angle\text{'s} \\
 \hline
 1,440 \\
 \hline
 10 \\
 \hline
 \boxed{144^\circ}
 \end{array}$$

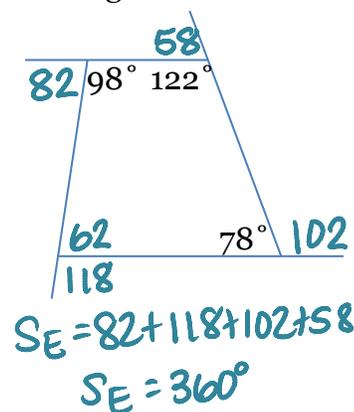
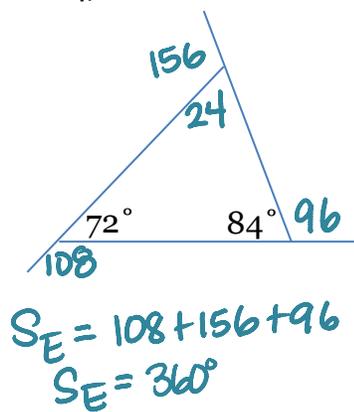
- 3) What is the name of the polygon in which the sum of the interior angle measures is 1800?

$$\begin{aligned}
 S_I &= (n-2)180 \\
 1800 &= (n-2)180
 \end{aligned}$$

$$\begin{aligned}
 10 &= n-2 \\
 +2 & \quad +2 \\
 \hline
 n &= 12
 \end{aligned}$$

Dodecagon

- 4) What about the exterior angles?



Polygon Exterior Angle Sum Theorem:

The sum of the exterior angle measures, one angle at each vertex, of a convex polygon is $S_E = 360^\circ$.

- 5) What is the sum of the exterior angles of an octagon?

$$\boxed{S_E = 360^\circ}$$

- 6) What is the measure of each exterior angle of an octagon?

$$S_E = 360$$

$$\text{Ext } \angle = \frac{360}{n} = \frac{360}{8}$$

- 7) An exterior angle measure of a regular polygon is given.

Find the number of sides and the measure of each interior angle.

$$72^\circ$$

$$E = \frac{360}{n}$$

$$72 = \frac{360}{n}$$

$$\boxed{n=5}$$

$$\frac{72n}{72} = \frac{360}{72}$$

$$\begin{array}{r}
 \text{Int.} = 180 - 72 \\
 \hline
 \boxed{= 108^\circ}
 \end{array}$$

$$\boxed{S_E = 45^\circ}$$

Please complete the chart.

Name of Polygon	Number of Sides (n)	Each Exterior Angle	Each Interior Angle	Sum of Ext. Angles	Sum of Interior Angles
Octagon	8	$\frac{360}{8} = 45^\circ$	$\frac{1080}{8}$ or $180 - 45$ 135°	360	$S_I = (n-2)180$ $1080 = (n-2)180$
Nonagon	9	$\frac{360}{9} = 40^\circ$	$180 - 40$ 140°	360°	$1,260^\circ$
Pentagon	$72 = \frac{360}{n}$ 5	$180 - 108$ 72°	108	360°	540°
Dodecagon	$30 = \frac{360}{n}$ 12	30	$180 - 30$ 150°	360°	$1,800^\circ$

1. What is a polygon?

2. Which polygons do you need to study the names of?

3. What is the difference between regular and irregular? Concave and convex?

4. How do you find the sum of the interior angles for a polygon?

$$S_I = (n-2)180^\circ$$

5. How do you find the sum of the exterior angles of a polygon?

$$S_E = 360^\circ$$

6. How do you find the measure of one interior angle of a regular polygon?

$$\text{Int } \angle = \frac{\text{Sum of Int } \angle\text{s}}{n} \quad \text{or} \quad \text{Int } \angle = 180 - \text{Ext } \angle$$

7. How do you find the measure of one exterior angle of a regular polygon?

$$\text{Ext } \angle = \frac{360}{n}$$

8. What is the relationship between an exterior and an interior angle?

Each Int \angle and ext \angle are supplementary!