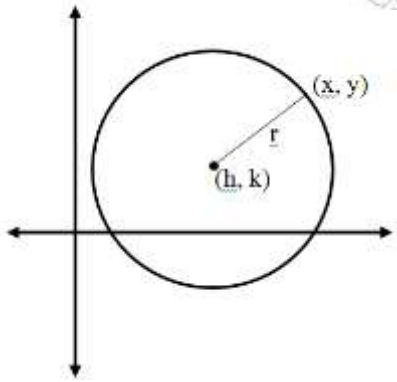
Key

12.7 Circles in the Coordinate Plane



The distance from the center to a point on the circle can be found using the distance formula:

Distance from (x, y) to (h, k) is:

$$r = \sqrt{(x-h)^2+(y-k)^2}$$

$$r^2 = (x-h)^2+(y-k)^2$$

This gives us the general formula for the equation of a circle.

Circle Equation: $(x-h)^2+(y-k)^2=r^2$

Center: (h,k) Radius: r

Ex 1: Find the equation of a circle whose center is (2, 7) and has a radius of 9.

$(x-2)^2+(y-7)^2=9^2$

$(x-2)^2+(y-7)^2=81$

Ex 2: Find the center and radius of $(x+3)^2+(y-2)^2=200$.

center: $(-3,2)$

$R=\sqrt{200}=2\cdot5\sqrt{2}=10\sqrt{2}$

Ex 3: Write an equation for the circle containing a center of (7, 5) and a point (3, -2).

$(x-7)^2+(y-5)^2=r^2$

$(3-7)^2+(-2-5)^2=r^2$

$(-4)^2+(-7)^2=r^2$

$16+49=r^2$

$65=r^2$

$(x-7)^2+(y-5)^2=65$

Ex 4: Graph: $(x-3)^2+(y+4)^2=9$

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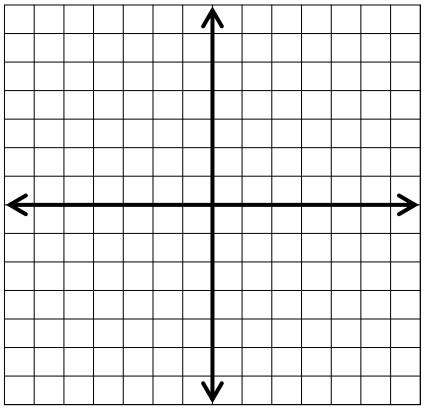


center $=(3,-4)$

$R=\sqrt{9}=3$

Key

Completing the Square

Warm-Up: Square the binomial.

1. $(x+3)^2$ $(x+3)(x+3)$ x^2+6x+9
2. $(x+4)^2$ $(x+4)(x+4)$ $x^2+8x+16$
3. $(x-7)^2$ $(x-7)(x-7)$ $x^2-14x+49$

Complete the Square: How can we reverse the procedure?
Find the number that "completes the square":

1. $x^2+4x+\underline{4}=(x+\underline{2})^2$ $\frac{4}{2}=2$ $2^2=4$
2. $x^2-16x+\underline{64}=(x-\underline{8})^2$ $\frac{-16}{2}=-8$ $(-8)^2=64$
3. $x^2+12x+\underline{36}=(x+\underline{6})^2$ $\frac{12}{2}=6$ $6^2=36$
4. $x^2-22x+\underline{121}=(x-\underline{11})^2$ $\frac{-22}{2}=11$ $(-11)^2=121$

To **change an equation of a circle** to the general form: $(x-h)^2+(y-k)^2=r^2$ where (h, k) is the center of the center and *r* is the radius.

Step 1: Arrange terms: $Ax^2+Bx+\_\_\_+Dy^2+Ey+\_\_\_=G$

Step 2: Find # to complete the square and add it to both sides

Step 3: Change to $(x-h)^2+(y-k)^2=r^2$ form

Example:

1. Complete the square to change into the general form. Then sketch the graph.

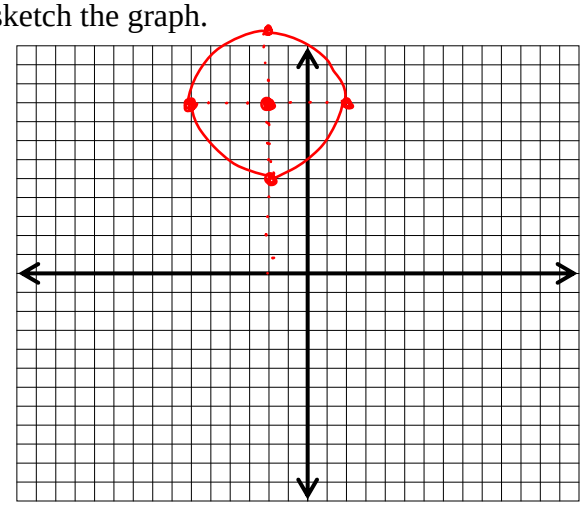
$x^2+y^2+4x-18y+69=0$

1) $x^2+4x+\underline{4}+y^2-18y+\underline{81}=-69+4+81$

2) $\frac{4}{2}=2$ $2^2=4$ $\frac{-18}{2}=9$ $(-9)^2=81$

3) $(x+2)^2+(y-9)^2=-69+4+81$

$(x+2)^2+(y-9)^2=16$ center $=(-2,9)$ $r=4$



2. Complete the square to change into the general form.

$x^2+y^2-6x-10y-65=0$

#1) $x^2-6x+\underline{9}+y^2-10y+\underline{25}=65+9+25$

$\frac{-6}{2}=-3$ $(-3)^2=9$ $\frac{-10}{2}=-5$ $(-5)^2=25$

3) $(x-3)^2+(y-5)^2=99$

center: $(3,5)$ $r=\sqrt{99}=3\sqrt{11}$

$99=9\cdot 11$

$3\cdot 3$