

# 9.4: COMPOSITIONS OF TRANSFORMATIONS

Target 9.4: Identify and draw compositions of transformations.

A composition of transformations is one transformation followed by another.

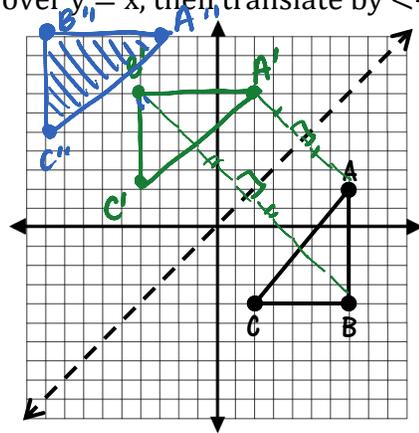
**Example 1:** Given  $\triangle ABC$ , A (7, 2), B (7, -4) and C (2, -4), Reflect over  $y = x$ , then translate by  $\langle -5, 3 \rangle$

Step 1: Reflect over  $y = x$

$\triangle A'B'C'$

Step 2: Translate by vector  $\langle -5, 3 \rangle$

$\triangle A''B''C''$



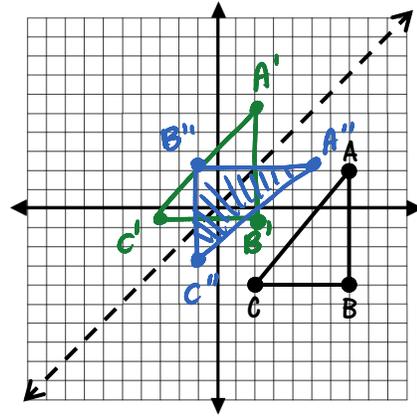
**Example 2:** Given  $\triangle ABC$ , A (7, 2), B (7, -4) and C (2, -4), Translate by  $\langle -5, 3 \rangle$ , then reflect over  $y = x$ .

Step 1: Translate by vector  $\langle -5, 3 \rangle$

$\triangle A'B'C'$

Step 2: Reflect over the  $y = x$

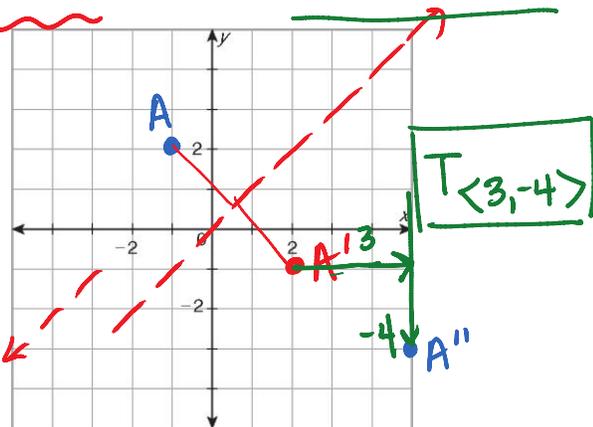
$\triangle A''B''C''$



**Think-About-It then Write-About-It!** Example 1 and 2 began with the same image. Both required you to perform the same transformations, but in a different order. What did you observe?

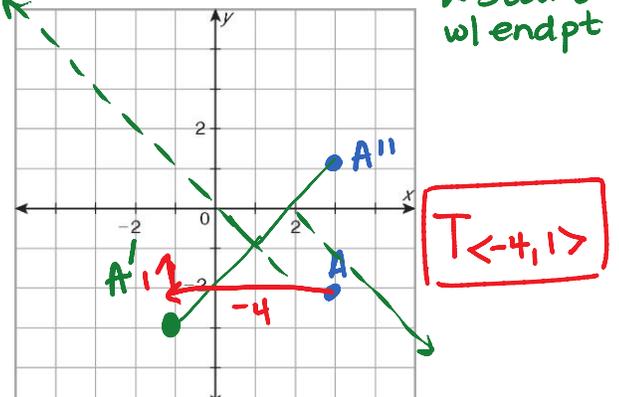
The final image ( $\triangle A''B''C''$ ) from Example 1 and 2 are  $\cong$ ; however, they did not end up in the same location. The order in which you transform the  $\triangle$ 's matters  $\text{☺}$  so be careful...

**Example 3:** Point A (-1, 2) was mapped to point A'' (5, -5) first by a reflection across the line  $y = x$ , and then by what translation vector? axis.



**Example 4:** Point A (3, -4) was mapped to point A'' (3, 1) first by an unknown vector  $\text{?!?}$  and then by a reflection across the  $y = -x$

Find the translation vector.



★ Start w/ endpoint

LET'S EXPLORE!

Part 1: Plot the points A(-6, 6), B(-4, 4), and C(0, 4).

1st: Reflect over  $y = 3$ . Label this figure  $\Delta A'B'C'$

2nd: Reflect  $\Delta A'B'C'$  over  $y = -1$ . Label this figure  $\Delta A''B''C''$ .

3rd: Write as a composition of isometries.

$$R_{y=-1} \circ R_{y=3}$$

4th: What single isometry would get us from the preimage ( $\Delta ABC$ ) to the final image ( $\Delta A''B''C''$ )?

a translation 8 units down

5th: Fill in the Blanks: A double reflection over ||

lines is a translation.

Notice anything else special?

The distance from preimage to image is twice the distance between the || lines

Part 2: Plot the points A(1, 0), B(7, 0), and C(3, -5)

1st: Reflect over  $x$ -axis. Label this figure  $\Delta A'B'C'$ .

2nd: Reflect  $\Delta A'B'C'$  over  $y$ -axis. Label this figure  $\Delta A''B''C''$ .

3rd: Write as a composition of isometries.

$$R_{y\text{-axis}} \circ R_{x\text{-axis}}$$

4th: What single isometry would get us from the preimage ( $\Delta ABC$ ) to the final image ( $\Delta A''B''C''$ )?

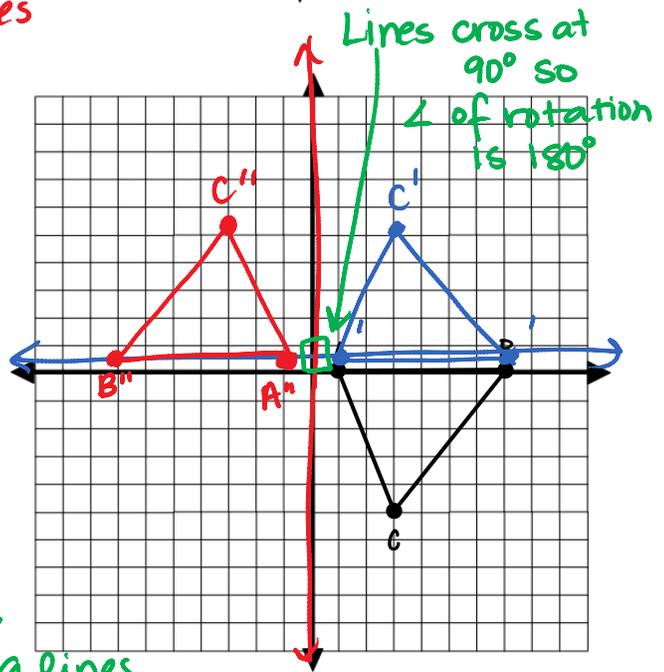
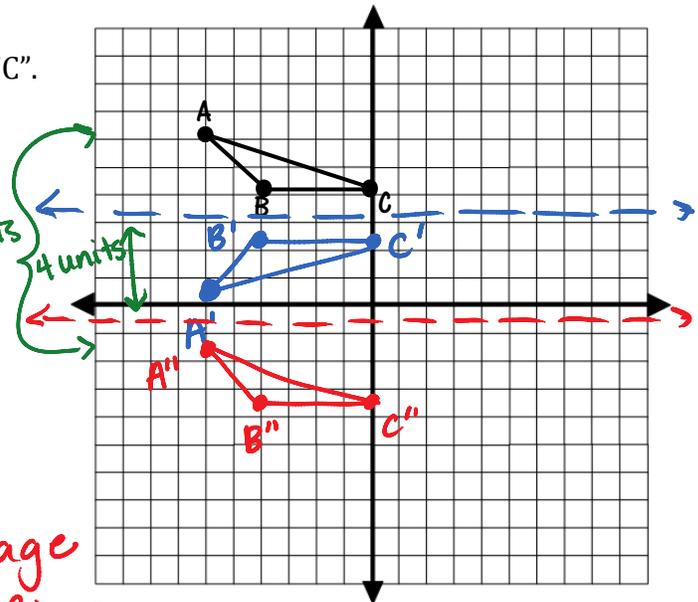
a  $180^\circ$  rotation

5th: Fill in the Blanks: A double reflection over  $\perp$

lines is a rotation.

Notice anything else special?

The angle of rotation is double the  $\angle$  measure created by the intersecting lines.



Example 5: Which transformation(s) would take Quadrilateral ABDC to Quadrilateral A'B'D'C'? For every choice you choose, provide the translation vector, line of reflection, and/or center of rotation.

~~a) Translation only~~

~~b) Reflection only~~

c) Rotation only *If so... about what pt?*

d) Translation and Reflection *Left 6 units, Reflect over x-axis*

$(-6, -3)$

