1-2 Points, Lines, and Planes

Common Core State Standards
G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment . . .
MP 1, MP 3, MP 4, MP 6

Objective To understand basic terms and postulates of geometry

Getting Ready!

Make the figure at the right with a pencil and a piece of paper. Is the figure possible with a straight arrow and a solid board? Explain.

Does how the arrow goes through the board make sense?

MATHEMATICAL PRACTICES In this lesson, you will learn basic geometric facts to help you justify your answer to the Solve It.

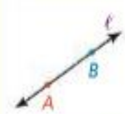
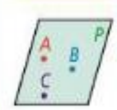
Essential Understanding Geometry is a mathematical system built on accepted facts, basic terms, and definitions.

Lesson Vocabulary
- point
- line
- plane
- collinear points
- coplanar
- space
- segment
- ray
- opposite rays
- postulate
- axiom
- intersection

In geometry, some words such as *point*, *line*, and *plane* are undefined. Undefined terms are the basic ideas that you can use to build the definitions of all other figures in geometry. Although you cannot define undefined terms, it is important to have a general description of their meanings.

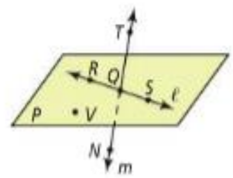
take note

Key Concept Undefined Terms

Term Description	How to Name It	Diagram
A **point** indicates a location and has no size.	You can represent a point by a dot and name it by a capital letter, such as A.	
A **line** is represented by a straight path that extends in two opposite directions without end and has no thickness. A line contains infinitely many points.	You can name a line by any two points on the line, such as $\overleftrightarrow{AB}$ (read "line AB") or $\overleftrightarrow{BA}$, or by a single lowercase letter, such as line ℓ.	
A **plane** is represented by a flat surface that extends without end and has no thickness. A plane contains infinitely many lines.	You can name a plane by a capital letter, such as plane P, or by at least three points in the plane that do not all lie on the same line, such as plane ABC.	

Points that lie on the same line are **collinear points.** Points and lines that lie in the same plane are **coplanar.** All the points of a line are coplanar.

Problem 1 Naming Points, Lines, and Planes



A What are two other ways to name $\overleftrightarrow{QT}$?

Two other ways to name $\overleftrightarrow{QT}$ are $\overleftrightarrow{TQ}$ and line m.

B What are two other ways to name plane P?

Two other ways to name plane P are plane RQV and plane RSV.

C What are the names of three collinear points? What are the names of four coplanar points?

Points R, Q, and S are collinear. Points R, Q, S, and V are coplanar.

Think

Why can figures have more than one name? Lines and planes are made up of many points. You can choose any two points on a line and any three or more noncollinear points in a plane for the name.

Got It? 1. a. What are two other ways to name $\overleftrightarrow{RS}$?

b. What are two more ways to name plane P?

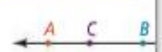
c. What are the names of three other collinear points?

d. What are two points that are *not* coplanar with points R, S, and V?

The terms *point*, *line*, and *plane* are not defined because their definitions would require terms that also need defining. You can, however, use undefined terms to define other terms. A geometric figure is a set of points. **Space** is the set of all points in three dimensions. Similarly, the definitions for *segment* and *ray* are based on points and lines.

take note

Key Concept Defined Terms

Definition	How to Name It	Diagram
A **segment** is part of a line that consists of two endpoints and all points between them.	You can name a segment by its two endpoints, such as $\overline{AB}$ (read "segment AB") or $\overline{BA}$.	
A **ray** is part of a line that consists of one endpoint and all the points of the line on one side of the endpoint.	You can name a ray by its endpoint and another point on the ray, such as $\overrightarrow{AB}$ (read "ray AB"). The order of points indicates the ray's direction.	
Opposite rays are two rays that share the same endpoint and form a line.	You can name opposite rays by their shared endpoint and any other point on each ray, such as $\overrightarrow{CA}$ and $\overrightarrow{CB}$.	

Plan

How do you make sure you name all the rays?
Each point on the line is an endpoint for a ray. At each point, follow the line both left and right to see if you can find a second point to name the ray.

Problem 2 Naming Segments and Rays

A What are the names of the segments in the figure at the right?

The three segments are $\overline{DE}$ or $\overline{ED}$, $\overline{EF}$ or $\overline{FE}$, and $\overline{DF}$ or $\overline{FD}$.

B What are the names of the rays in the figure?

The four rays are $\overrightarrow{DE}$ or $\overrightarrow{DF}$, $\overrightarrow{ED}$, $\overrightarrow{EF}$, and $\overrightarrow{FD}$ or $\overrightarrow{FE}$.

C Which of the rays in part (B) are opposite rays?

The opposite rays are $\overrightarrow{ED}$ and $\overrightarrow{EF}$.

Got It? 2. Reasoning $\overrightarrow{EF}$ and $\overrightarrow{FE}$ form a line. Are they opposite rays? Explain.

A **postulate** or **axiom** is an accepted statement of fact. Postulates, like undefined terms, are basic building blocks of the logical system in geometry. You will use logical reasoning to prove general concepts in this book.

You have used some of the following geometry postulates in algebra. For example, you used Postulate 1-1 when you graphed equations such as $y = 2x + 8$. You graphed two points and drew the line through the points.

take note

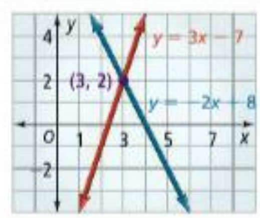
Postulate 1-1

Through any two points there is exactly one line.

Line t passes through points A and B. Line t is the only line that passes through both points.

When you have two or more geometric figures, their **intersection** is the set of points the figures have in common.

In algebra, one way to solve a system of two equations is to graph them. The graphs of the two lines $y = -2x + 8$ and $y = 3x - 7$ intersect in a single point (3, 2). So the solution is (3, 2). This illustrates Postulate 1-2.

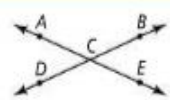


take note

Postulate 1-2

If two distinct lines intersect, then they intersect in exactly one point.

$\overleftrightarrow{AE}$ and $\overleftrightarrow{DB}$ intersect in point C.



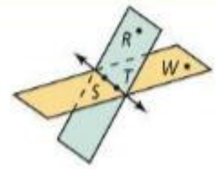
There is a similar postulate about the intersection of planes.

take note

Postulate 1-3

If two distinct planes intersect, then they intersect in exactly one line.

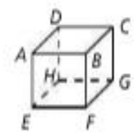
Plane RST and plane WST intersect in $\overleftrightarrow{ST}$.



When you know two points that two planes have in common, Postulates 1-1 and 1-3 tell you that the line through those points is the intersection of the planes.

Problem 3 Finding the Intersection of Two Planes

Each surface of the box at the right represents part of a plane. What is the intersection of plane ADC and plane BFG?

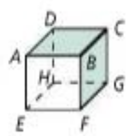


Know
Plane ADC and plane BFG

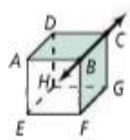
Need
The intersection of the two planes

Plan
Find the points that the planes have in common.

Think
Is the intersection a segment?
No. The intersection of the sides of the box is a segment, but planes continue without end. The intersection is a line.



Focus on plane ADC and plane BFG to see where they intersect.



You can see that both planes contain point B and point C.

The planes intersect in $\overleftrightarrow{BC}$.

Got It? 3. a. What are the names of two planes that intersect in $\overleftrightarrow{BF}$?

b. Reasoning Why do you only need to find two common points to name the intersection of two distinct planes?

When you name a plane from a figure like the box in Problem 3, list the corner points in consecutive order. For example, plane $ADCB$ and plane $ABCD$ are also names for the plane on the top of the box. Plane $ACBD$ is not.

Photographers use three-legged tripods to make sure that a camera is steady. The feet of the tripod all touch the floor at the same time. You can think of the feet as points and the floor as a plane. As long as the feet do not all lie in one line, they will lie in exactly one plane.

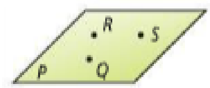
This illustrates Postulate 1-4.

take note

Postulate 1-4

Through any three noncollinear points there is exactly one plane.

Points Q, R, and S are noncollinear. Plane P is the only plane that contains them.

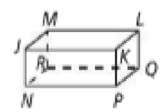


Problem 4 Using Postulate 1-4

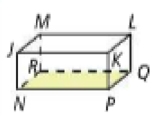
Plan

How can you find the plane?
Try to draw all the lines that contain two of the given points. You will begin to see a plane form.

Use the figure at the right.

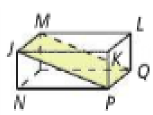


A What plane contains points N, P, and Q? Shade the plane.



The plane on the bottom of the figure contains points N, P, and Q.

B What plane contains points J, M, and Q? Shade the plane.



The plane that passes at a slant through the figure contains points J, M, and Q.

Got It? **4. a.** What plane contains points L, M, and N? Copy the figure in Problem 4 and shade the plane.

b. **Reasoning** What is the name of a line that is coplanar with $\overleftrightarrow{JK}$ and $\overleftrightarrow{KL}$?