5.7 The Pythagorean Theorem

Simplifying Radicals: We have to rewrite square roots to be in simplest form just like we have to reduce fractions. Sometimes they simplify just to an integer, while other times they reduce into a number and a square root. In order to do this, it is usually easier to look for perfect squares. Perfect squares are numbers like: 1 (1 x 1), 4 (2 x 2), 9 (3 x 3).

Directions: Please list the first 15 perfect squares.

1 4 9 16 25 36 49 64 81 100 121 144 169 196 225

Directions: Simplify the following radicals: (no decimals)

$\sqrt{36}$ a perfect square! 6

$\sqrt{16}$ 4

$\sqrt{49}$ 7

$\sqrt{121}$ 11

$\sqrt{196}$ 14

Directions… Take it up a Notch! Simplify the following radicals. (no decimals)

$\sqrt{12}$
· not a perfect square ☹
· try to find a perfect square…
$\sqrt{4}\cdot 3$
$2\sqrt{3}$

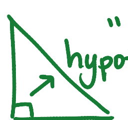
$\sqrt{72}$
opt 1: $\sqrt{72}$, 9 8, 4 2, $\sqrt{9\cdot 4}\,2$, $3\cdot 2\sqrt{2}$, $6\sqrt{2}$
opt 2: $\sqrt{72}$, 36 2, $\sqrt{36}\,2$, $6\sqrt{2}$

$\sqrt{48}$
$\sqrt{16}\cdot 3$
$4\sqrt{3}$

$\sqrt{80}$
4 20
4 5
$\sqrt{4\cdot 4}\,5$
$4\sqrt{5}$

$\sqrt{75}$
25 3
$\sqrt{25\cdot 3}$
$5\sqrt{3}$

RIGHT TRIANGLE ANATOMY

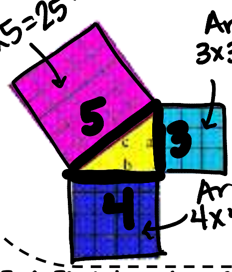


a = leg } can be switched!
b = leg }
c = hypotenuse – always opposite 90° ∠

We can use the Pythagorean Theorem to find a missing side length in a right triangle when we already know two side lengths. In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

http://www.mathsisfun.com/pythagoras.html

Visually this can be seen as…

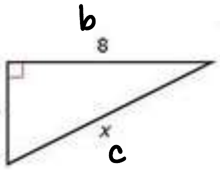


9+16 = 25
25 = 25 ✓
Awesome ☺

Algebraically this can be written as…

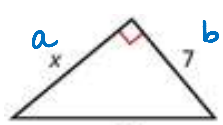
$a^2 + b^2 = c^2$
$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$ ✓
woo hoo!

Ex 1: Find the value of x. Leave answer as reduced radical.



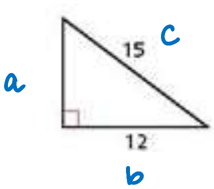
$a^2 + b^2 = c^2$
$4^2 + 8^2 = c^2$
$16 + 64 = c^2$
$\sqrt{80} = \sqrt{c^2}$
$\sqrt{16}\,5 = c$
$4\sqrt{5} = c$

Ex 2: Find the value of x. Leave answer as reduced radical.



$a^2 + b^2 = c^2$
$x^2 + 7^2 = 11^2$
$x^2 + 49 = 121$
$x^2 = 72$
$x = \sqrt{72}$
9 8
4 2
$x = \sqrt{9\cdot 4}\cdot 2 = 6\sqrt{2}$

Ex 3: Find the value of x.



$a^2 + b^2 = c^2$

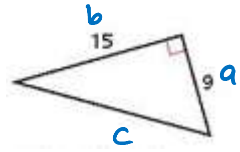
$a^2 + 12^2 = 15^2$

$a^2 + 144 = 225$

$a^2 = 81$

$\boxed{a = 9}$

Ex 4: Find the value of x. Leave answer as reduced radical.



$a^2 + b^2 = c^2$

$9^2 + 15^2 = c^2$

$81 + 225 = c^2$

$306 = c^2$

$\sqrt{306} = c$

9 34

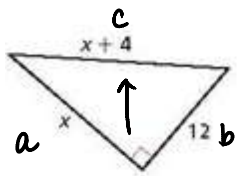
2 17

$\sqrt{9 \cdot 2 \cdot 17} = c$

$\boxed{3\sqrt{34} = c}$

Take it up a Notch!

Ex 5: Find the value of x.



$a^2 + b^2 = c^2$

$x^2 + 12^2 = (x+4)^2$

$x^2 + 144 = (x+4)(x+4)$

$\cancel{x^2} + 144 = \cancel{x^2} + 8x + 16$

$128 = 8x$

$\boxed{16 = x}$ ← woohoo!

Check: $a = 16$ $b = 12$ $c = 16 + 4 = 20$

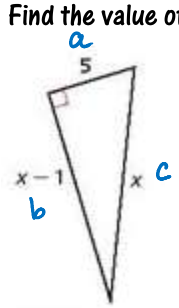
So $a^2 + b^2 = c^2$

$16^2 + 12^2 \stackrel{?}{=} 20^2$

$256 + 144 = 400$

$400 = 400$

Ex 6: Find the value of x.



$a^2 + b^2 = c^2$

$5^2 + (x-1)^2 = x^2$

$25 + (x-1)(x-1) = x^2$

$25 + \cancel{x^2} - 2x + 1 = \cancel{x^2}$

$26 - 2x = 0$

$26 = 2x$

$13 = x$

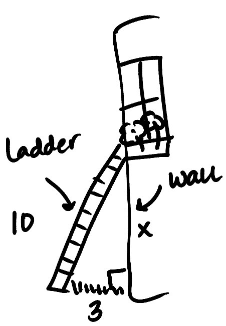
Check: $a = 5$ $b = 12$ $c = 13$

$5^2 + 12^2 \stackrel{?}{=} 13^2$

$169 = 169$ ✓

Real Life Applications ☺

Ex 7: Patrick needed to paint the windowsill. He placed a 10 foot ladder 3 feet away from the wall. Will the ladder reach the windowsill if it is 9.8 feet above the ground? Why or why not? (round to the nearest tenth)



$a^2 + b^2 = c^2$

$a^2 + 3^2 = 10^2$

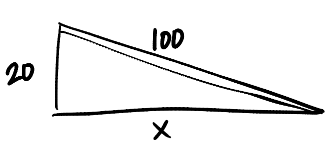
$a^2 + 9 = 100$

$a^2 = 91$

$a \approx 9.5$ ft

The Ladder will be just short of reaching the window since the Ladder will reach up to 9.5 ft and the windowsill is 9.8 ft.

Ex 8: Given a 100 ft. long ramp that is constructed from the top of a 20 ft. wall to ground level, find the distance along the ground from the wall to the end of the ramp (to the nearest hundredth).



$a^2 + b^2 = c^2$

$20^2 + x^2 = 100^2$

$400 + x^2 = 10{,}000$

$x^2 = 9{,}600$

$x \approx 97.98$ feet

The distance from the wall to the end of the ramp is approx. 97.98 ft

What if it is not a right triangle? Or is it?!

Flashback…

In order to determine whether or not a figure can be a triangle, we have to check that the sum of the two smaller sides is greater than the 3rd side. $\underbrace{a+b}_{\text{2 smaller sides}} > c$ ↖ Longer side

Flash-forward…

Pythagorean Inequalities Theorem: If ΔABC, c is the length of the Longest side.

If $c^2 = a^2 + b^2$, then ΔABC is a right triangle.

If $c^2 > a^2 + b^2$, then ΔABC is an obtuse triangle.

If $c^2 < a^2 + b^2$, then ΔABC is an acute triangle.

Directions: Tell if the measures can be side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

Ex 1: 5, 7, and 10 (c)

1st Is it a Δ? $a+b>c$

$5+7>10$

$12>10$ ✓

$\boxed{\text{YES}}$ ☺

2nd Type of Δ: $c^2 ? a^2+b^2$

$10^2 ? 5^2+7^2$

$100 ? 25+49$

$100 > 74$

$\boxed{\text{obtuse } \Delta}$

Ex 2: 8, 11, and 13

1st Δ? $8+11>13$

$19>13$ ✓

Yes!

2nd Type: $13^2 ? 8^2+11^2$

$169 ? 64+121$

$169 < 185$

Acute Δ

Ex 3: 5, 8, and 17

1st Δ? $5+8>17$

$13 \not> 17$ no

not a Δ

Ex 4: 7, 10, and 12

1st Δ? $7+10>12$

$17>12$ ✓

Yes

2nd Type: $12^2 ? 7^2+10^2$

$144 ? 49+100$

$144 < 149$

Acute Δ

On a scale of 1 to 5, where do you rate today's lesson?	Yikes!		Kind of got it…		I can do it in my sleep!!!
Simplifying Radicals:	1	2	3	4	5
Pythagorean Theorem:	1	2	3	4	5
Converse of the Pythagorean Theorem:	1	2	3	4	5

Sad but true ☺